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Aerodynamic Aspects of the Magnetospheric Flow

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I. Introduction

It has been apparent for some time that the solar corona should be considered as extending well beyond the orbit of the earth. Diverse observations and interpretations that have contributed to our understanding of this and related phenomena include the scattering of sunlight, occultation of radio stars, behavior of comet tails, earth-sun phenomenological correlations, modulation of galactic cosmic rays, modulation of energetic particles of solar origin, and, more recently and definitively, direct satellite observations. The over-all picture one has is that the corona is heated at its base (probably by absorbing noise generated in lower turbulent levels of the sun) and expands freely into the solar system. This general outward flow is often referred to as the solar wind. Near the orbit of the earth the flow is supersonic, having a Mach number of 7–10. The solar wind consists almost entirely of fully ionized hydrogen.

The modulation of the forementioned cosmic rays is clear proof of the existence of a magnetic field in the solar wind; the strength of this field is on the order of 5γ (5×10^{-5} gauss). This is, of course, far less than the magnetic field near the surface of the earth which is about 0.5 gauss. Both these magnetic fields influence the motions of energetic cosmic rays, the former because of its greater extent; both fields also influence the motions of the less energetic but more numerous particles making up the solar wind. The latter particles, however, carry almost all the mass, momentum, and energy resident in the interplanetary gas. It is therefore more accurate to say that the over-all magnetic field configuration will be determined by the mechanics of these low-energy particles. The orbits of higher-energy particles will in turn be determined by the magnetic and electric fields resulting from this flow.

Two regions in the configuration resulting from the interaction of the solar wind with the earth's magnetic field have

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been recognized for some time. The magnetosphere boundary, which separates these regions, is defined as the boundary to which the solar-wind particles can penetrate before being stopped by the earth's field. Considerable attention has been directed to the analysis of the flow external to the magnetosphere and the determination of the shape of the boundary. On the other hand, ionospheric and auroral observations strongly suggest that a significant flow pattern also exists inside the magnetosphere. Dungey[1] has observed that the ionospheric current pattern is consistent with the one that would result if a significant fraction of the interplanetary magnetic field lines became reconnected to the magnetic field lines from the earth. The resulting component of magnetic field normal to the boundary provides a handle by which the solar wind can drag the magnetic field lines within the magnetosphere. Dungey[1] was therefore led to suggesting that a mechanism exists which gives rise to appreciable reconnection of the field lines at the magnetosphere boundary. Hines and Axford[2] have greatly extended the model of internal convection patterns within the magnetosphere. Although they do not specify the driving mechanism in detail, they suggest that it is provided by a shear stress at the magnetosphere boundary. More recently Petschek[3] has analyzed the flow in the neighborhood of a boundary across which there is a sharp change in magnetic field direction and showed that the resulting rate of field cutting decreases only logarithmically with the magnetic Reynolds number. The logarithmic dependence implies a rapid rate of reconnection which fortunately is very insensitive to uncertainties in the effective conductivity of the plasma. As will be shown below, applying this result to the flow over the magnetosphere, we conclude that more than 10% of the magnetic field lines brought to the surface of the magnetosphere by the solar wind get broken and reconnected to the earth's polar field lines. This rapid rate of reconnection is in rough quantitative agreement with the amount required to drive the internal convection pattern. The combination of the analyses of the external, internal, and boundary region flows thus provides us with a general view of the over-all flow pattern. This description provides some insight into the extent to which phenomena occurring within the magnetosphere and in the auroral regions are related to the solar wind.

In Secs. III, IV, and V the three component regions will be reviewed, and in Sec. VI the results will be synthesized. Before proceeding with this program, we must, however, define what basic description of the plasma is appropriate. On the basis of the fact that the mean free path for particle scattering by binary collisions is large, many authors have attempted to describe the flow in terms of the individual particle trajectories. On the other hand, others have treated the fluid as a continuum and applied the magnetohydrodynamic equations. In Sec. II we will review the arguments that suggest that the continuum rather than the particle trajectory approach is more appropriate for the consideration of the basic flow.

The major objective of this paper is to describe the over-all features of the flow of the solar wind over the magnetosphere. This description is undertaken with a view to synthesizing the experimental observations with theoretical considerations to provide an intelligible view of the whole. However, the multiplicity of relevant phenomena and the complicated character of many of the observations mean that any attempt at concise description must be strongly selective both in terms of the evidence brought out and theoretical interpretation. In the present paper, the authors have deliberately tried to give a coherent explanation for selected phenomena rather than an encyclopedic review of facts and theories concerning the magnetosphere.

II. Justification of Continuum Assumption

Although the conditions in the plasma composing the solar wind are known to fluctuate appreciably, the data from Mariner[4] have shown that the solar wind exists at all times. Commonly accepted typical conditions in the solar wind correspond to a magnetic field of the order of 5γ, a particle density of about 10 protons/cm^3, flow velocities of the order of 500 km/sec, and thermal velocities of the ions of about $\frac{1}{10}$ the flow velocity. We may note that this corresponds to a supersonic flow since the flow velocity is about 10 times either the thermal velocity or the Alfven speed. Other basic parameters that may be deduced from the foregoing plasma conditions are that the Debye length is of the order of 10 m, the ion gyroradius based on this magnetic field and the streaming velocity is of the order of 10^3 km, and the mean free path for Coulomb scattering based on the thermal velocity is of the order of one a.u.* and would be appreciably larger if based on the streaming velocity. The radius of the magnetosphere is about ten earth radii or roughly 10^5 km.

It is apparent that the mean free path for binary collisions is larger by several orders of magnitude than the entire magnetosphere. If we were dealing with the flow of an unionized gas, we would immediately conclude that a continuum approach was unjustified and that a free molecule approach should be used. In the case of a plasma, however, the situation is fundamentally different, since there are several mechanisms involving electric and magnetic fields which give rise to coherent phenomena in a plasma. For example, we cannot have appreciable differences between the electron and ion densities over distances larger than the Debye length, which is minute compared to the scale of the magnetosphere. A further example of coherence is the known existence in collision-free plasma of several linear wave propagation modes (plasma oscillations, whistlers, etc.). It is therefore clear that the orbits of individual particles are much more dependent on the orbits of other particles than they would be in the case of free molecule flow of an unionized gas. More specifically, it can be shown that in two special cases the isentropic continuum magnetohydrodynamic equations apply to a collision-free plasma. The first of these is the case in which the ion thermal velocities are small compared to either the flow velocity or the Alfven speed (this is often referred to as the cold plasma case). This case will be discussed below. In the second case, finite temperatures are allowed, but we restrict ourselves to a two-dimensional flow in which the magnetic field is perpendicular to the plane of the flow. In this case it can be shown that the conservation of the magnetic moments of the individual particles gives rise to a pressure-density relation that is completely equivalent to the usual isentropic relation corresponding to a gas with two degrees of freedom.[5] As a result, as long as flow properties change only by small amounts in a distance of the order of the gyroradius, detailed consideration of the particle orbits does in fact lead us to the isentropic continuum magnetohydrodynamic equations. Thus, in this particular case we are able to use a continuum treatment to describe even nonlinear effects. It follows almost immediately from this conclusion that a compression pulse will steepen to form a discontinuity or shock wave.[6] Since the steepening analysis is valid for gradients small compared to the gyroradius, the steepening process will continue at least until thicknesses of the order of the ion gyroradius are reached. It also has been shown that a nonlinear pulse propagating along the magnetic field lines into a cold plasma will steepen to form a discontinuity.[7] We are thus led to the conclusion that in at least these two special cases coherent effects in a collision-free plasma can lead to the familiar aerodynamic concept of the formation of shock waves and that these shock waves may indeed have a thickness that is small compared to the relevant flow dimensions for the magnetosphere. There have been several attempts to give theoretical descriptions of the structure of such collision-free shock waves.[8–14] There is

* The a. u. (astronomical unit) is a unit of distance equal to the radius of the orbit of the earth, that is, 1.5×10^8 km.

considerable disagreement as to the mechanisms that are responsible for controlling the shock structure and even the order of magnitude of the shock thickness. However, none of these theories predicts shock thicknesses for strong shocks that are more than a few times the ion gyroradius. Thus, on the basis of any of them we would expect shock waves that are thin compared to the dimensions of the magnetosphere. A further feature to which we will return below and which all of hese theories have in common is that, in the shock front itself and for some distance behind it, the flow is highly nonuniform and might be described as being turbulent.

Unfortunately, there is at present no clear-cut laboratory experimental evidence for the existence of collision-free shock waves. Some experiments that had been identified as exhibiting collision-free shocks[13, 14] may be subject to other interpretations.[15,16] There are also no published results from satellite experiments which demonstrate clearly the existence of collision-free shock waves in the interplanetary medium. Very recent satellite experiments may answer this question unambiguously.† At the present time the best evidence for the existence of collision-free shock waves comes from terrestrial observations of the sudden commencement of magnetic storms. These are events in which a day or two after a solar flare almost all magnetic observatories register a sharp increase in magnetic field with a rise time of the order of 2 min. The rapid rise time of the disturbance as compared to the long travel time from the sun to earth clearly indicates coherent behavior of the plasma. However, if we calculate a velocity for the disturbance based on the time it takes to come from the sun, we find that 2 min is close to the time taken for the disturbance to pass the magnetosphere. Thus, even an infinitely thin shock wave would give rise to a terrestrial disturbance having a rise time of at least 2 min. Consequently, this observation does not allow us to set a lower limit to the thickness of the disturbance.

Thus, on theoretical grounds with some experimental backing, we would expect some phenomena characteristic of continuous flows, in particular the formation of shock waves, to exist even in a collision-free plasma. At this point, therefore, let us attempt to see what assumptions are required to arrive at the full set of continuum equations starting from the known equations that govern the individual particle motions. The individual particle motions can be described in terms of the Boltzmann equation neglecting binary collisions. We may now, with complete rigor, take the moments of the Boltzmann equation corresponding to the conservation of mass, momentum, and energy.[18] The resulting equations are similar to the magnetohydrodynamic equations with the exception that the pressure and energy flux are tensors defined only in terms of moments over the distribution of particle velocities. Since, in order to define these tensors, we would have to analyze the particle orbits in detail, the procedure of taking moments is at first sight somewhat useless. If, however, we can find an assumption sufficient to define these moments over the particle distributions, then the moment equations would be of some value. This is particularly easy to do in the case of a plasma with zero temperature (i.e., zero thermal velocity). In this case, the distribution function becomes a delta function in velocities, i.e., at a particular point in space and time all of the particles have precisely the same velocity. It then is easy to take the moments of the distribution function, and the moment equations reduce to the continuum hydrodynamic equations for a zero temperature plasma. This particular case is of some interest. It can be shown, for example, that compression waves traveling in arbitrary directions relative to the magnetic field into a zero temperature plasma, will steepen to form shock waves. We cannot, however, expect this case to apply in the flow over the magnetosphere where the presence of strong shock waves will give rise to high temperatures.

In ordinary hydrodynamics when the mean free path is small compared to the scale of the flow field, we are justified in assuming that the particle distributions are randomized by collisions and, therefore, that to zero order the particle distribution is isotropic. This assumption leads us to the isentropic continuum equations. First-order corrections in terms of the ratio of mean free path to scale length of the flow give rise to slight distortions of the distribution and result in the transport terms (viscosity and heat conduction) in the hydrodynamic equations. In the case of the flow over the magnetosphere, we will also make the assumption that the distance in which particle motions are randomized is small compared to the flow dimensions. This assumption, of course can not be justified in terms of scattering by binary collisions since the mean free path for this scattering is large compared to the magnetosphere. It may, however, be possible to justify it in terms of the observed turbulence that exists just outside the boundary of the magnetosphere. Measurements from Pioneer I indicated that the magnetic field changes by its own order of magnitude with characteristic frequencies in the neighborhood of 1 rad/sec.[19] Appreciable fluctuations at higher frequencies also may be present, but these would have been beyond the frequency response of this particular experiment.

We note that this frequency is about half of the cyclotron frequency of the ions in the average magnetic field and that, as the magnetic field fluctuates, so does the cyclotron frequency. This suggests that the ion motions will be affected by the cyclotron resonance. Of course, the opposite effect in which the particle motions affect the field is also present. These remarks suggest that there is an effective randomizing time for the particles which corresponds very roughly to the angular frequency of the magnetic field turbulence and to the ion cyclotron frequency ~ 1 sec. Another way of putting this is to say that the randomizing distance or equivalent mean free path is the distance a particle travels in a cyclotron period, or what is the same thing, a Larmor radius. These distances amount to a few hundred kilometers and are indeed small compared to the dimensions of the magnetosphere flow. This should not be taken too seriously as a quantitative estimate of the effective dissipation length. It does, however, make plausible the assumption that the randomizing length is small compared to the flow dimensions, and, therefore, there is some justification in using continuum equations to describe the flow field. It should be pointed out that from this viewpoint the transport coefficients will depend critically on the effective dissipation length or mean free path and, therefore, that our knowledge of the effective value of these transport coefficients has considerable uncertainty. As we shall see later, certain features of the magnetosphere flow do depend in principle on the transport coefficients, but are nevertheless highly insensitive to their magnitudes. Thus, the results that we will describe will be appreciably more accurate than our knowledge of the transport coefficients.

Given the turbulence in the magnetic field, the calculation of detailed particle trajectories becomes effectively impossible. As a result, calculations based on the study of detailed trajectories have neglected the turbulence and have replaced the actual magnetic field by a smooth one. This assumes that the particles can negotiate the fluctuating field without appreciably modifying their orbits. Since several of the continuum features of a collision-free plasma could be derived without the assumption of turbulent scattering, it is probable that many of the features of the flow derived from these two view-points will be similar. However, it would seem that, in the cases where the results do not agree, the continuum assumption has a greater a priori probability of

† At the time this paper was written (December 1963), highly preliminary results from plasma probe measurements on IMP (Explorer XVIII), which had then been in orbit only one week, appeared to give evidence for a thin shock.[17] Since then, a large amount of data obtained by this important satellite have been released.[48,49] These data are briefly described in the Appendix to this paper, written in August 1964.

being correct than the assumption that the turbulence in the plasma can be neglected.

We should point out that the continuum approach will not describe some of the features of the flow. In particular, the magnitude of the turbulent fluctuations will not be described, and we will consider only the average properties. Also, although we have said that the moments over the distribution function can be approximated as being almost symmetric, this should not be taken to imply that there is not a small fraction of the particles with, for example, very high energies as compared to the average particle energy. To obtain the turbulent amplitudes or the detailed particle distribution, more detailed analysis would be required. Thus, with the present analysis we limit ourselves to describing the gross properties of the flow field.

III. External Flow

The original suggestion that there should be a termination to the earth's magnetic field because of the surrounding plasma is due to Chapman and Ferraro.[20] They envisioned a stream of particles coming from the sun in the absence of an interplanetary magnetic field. As the particles impinge on the earth's field, they are deflected. The force required to reflect the entire stream of particles may be considered as resulting from the $j \times B$ force associated with the motion of the individual particles. Since the gyroradii of the particles are small, the region in which they turn and, therefore, the region in which the current exists is very narrow. Also, outside of this region there can be no magnetic field, otherwise the particles would be deflected there. This therefore, leads to a very abrupt termination of the earth's magnetic field.

The shape of the boundary can be determined by the condition that the force required to reflect the particles from the boundary is $2\rho u^2 \cos^2\theta$, where ρ is the density, u the stream velocity, and θ the angle between the stream direction and the normal to the boundary surface. This force must be equal to the magnetic force on the boundary which in turn is the magnetic pressure just inside the boundary $B^2/8\pi$. The problem of determining the shape of the boundary then reduces to the problem of solving for the magnetic field in the vacuum inside the cavity, with the boundary conditions that the earth's dipole field exists near the origin and that the shape of the cavity must be such as to satisfy the pressure condition as just stated. In the hypothetical two-dimensional case, this can be done precisely by conformal mapping techniques.[21, 22] In three-dimensions, it is not as yet possible to obtain an analytical solution for the internal magnetic field, and therefore several approximations have been used to estimate the magnetic field just inside the boundary.[23–25] The most common of these has been to assume that just inside the boundary the magnetic field has a value that is twice what the tangential component would have been for a pure dipole field.[24, 26, 27] The general shape of the boundary, which is obtained in this manner, is that in the solar direction the boundary occurs at about 10 earth radii. Its radius of curvature in this region is slightly larger than its distance from the earth. As one goes around behind the earth, the cavity approaches a constant width. The cavity would close at a finite distance behind the earth if one allowed for some thermal motion in the freestream.

It should be noted that the details of the shape are relatively insensitive to variations in the calculation,[28] since the dipole magnetic field falls off as the cube of the radius and therefore the magnetic pressure as the sixth power. The distance to any point on the cavity therefore only changes with the sixth root of the changes in, for example, density of the stream. For this reason, if we look at the same problem from the continuum viewpoint, we will find very little difference in the shape of the cavity; however, the flow outside of the cavity will be appreciably altered. As long as the medium is assumed to have a high electrical conductivity, the flow will still not be able to penetrate into the earth's field. As a result, the cavity can be represented as a blunt body in a hypersonic flow. In this case we expect a bow shock some distance ahead of the body or magnetosphere boundary.[29–31] The region between the shock and the body would then contain a high-temperature plasma as well as appreciable turbulence generated by the shock.

The fact that the shape of the magnetosphere boundary will not be altered significantly is seen most easily by noting that a rough approximation to the pressure distribution on a hypersonic body is the Newtonian distribution. The boundary condition at the surface is that the pressure just outside must equal the magnetic pressure inside. The Newtonian distribution is, apart from a factor of 2, identical to the pressure required to reflect the particles at the boundary. Therefore, the shape obtained from the free-particle picture is identical to the shape obtained from the Newtonian distribution if the freestream density is twice as high in the Newtonian case. Several refinements of the aerodynamics beyond the Newtonian distribution are, of course, possible, based on our presently more detailed knowledge of the hypersonic flow over blunt bodies.[32] These would produce some quantitative changes in the shape, particularly away from the stagnation region. However, details some distance behind the earth are probably highly questionable. An unsteady wake could, for example, cause the downstream portion of the magnetosphere cavity to oscillate. Also, as will be discussed in Sec. VI, there is some reason to believe that field reconnection can appreciably affect the shape of the downstream portions of the cavity.

In our discussion thus far, we have neglected the effect of the interplanetary magnetic field on the aerodynamic calculation, except, of course, that we have made use of it to justify the continuum approach. This is a reasonable approximation in much of the flow. The dynamic pressure in the interplanetary plasma is so much larger than the magnetic pressure that behind a normal shock the gas pressure will be several times the magnetic pressure, and, therefore, it seems reasonable to neglect the magnetic pressure. There is, however, at least one exception to this which occurs in the neighborhood of the stagnation point. If we look at a tube of magnetic field lines which is moving toward the stagnation point, the plasma on this field tube can flow out along the field lines. The magnetic field can, however, only go around the body by moving sideways. For field lines close to the stagnation streamline, the plasma can escape more rapidly than the field lines. Thus, we may expect that the density would decrease as the stagnation point is approached. The decrease in plasma pressure would be balanced by an increase in the magnetic pressure. This conclusion may explain the observation that, in some cases, although the direction of the magnetic field changes abruptly at the magnetosphere boundary, its magnitude is almost the same on both sides.[33]

In summary, the general shape of the magnetosphere boundary is a blunt body that extends considerably further behind the earth than it does toward the sun. The quantitative description is quite accurate near the stagnation point, but uncertainties grow as we move toward the region behind the earth. We would expect a bow shock at nearly the ordinary aerodynamic standoff distance ahead of the stagnation point.

Satellite experiments have confirmed the existence of a sharp magnetosphere boundary that occurs at approximately the expected location in the neighborhood of the stagnation point.[33] Behind the earth the boundary has also been observed.[34, 35] Its position in this region seems to vary with time. It is not clear whether this is caused by variations in the strength and direction of the solar wind or by unsteadiness in the wake. In any event, observations of the magnetic field strength in this region are in conflict with the simple aerodynamic model. This point will be discussed in Sec. VI.

The region between the boundary and the shock has been shown to contain appreciable turbulence[19] as was to some extent anticipated by collision free shock theories. As mentioned earlier, the shock wave itself may have been seen on very recent experiments.[17]

IV. Internal Motions

It was observed by Gold[36] that the resistance of the earth's atmosphere would permit motion within the magnetosphere. Since the earth is a good conductor, the magnetic field lines are solidly anchored at ground level. If there were direct contact with the conducting plasma in the magnetosphere, the base of the magnetosphere would have to move with the earth. However, with the insulation provided by the atmosphere, the field and, therefore, the base of the magnetosphere are free to slide over the top of the ionosphere. This situation is quite analogous to the case of a copper disk placed between the pole pieces of a magnet. If the disk rotates, it will become polarized and have a radial electric field within it. If no electrical connection is made to a stationary conductor, no current will flow, and there will be no deceleration of the angular velocity of the disk. The electric field within the copper would be such that $E + v \times B = 0$. This is just the condition that, in plasma language, is described as having the magnetic field lines within the copper moving with the copper. We thus have a situation in which the magnetic field in the copper disk (magnetosphere) moves relative to the magnetic field in the pole face (earth).

The motions within the magnetosphere are, however, severely restricted. Since the dipole field increases rapidly in magnitude as we move in from the boundary, we would expect the magnetic pressure to be large compared to the plasma pressures over most of the magnetosphere. As a result, the magnetic field must be very accurately force-free. This implies that in these regions the field will be very close to the dipole field. Motions that are allowed must, therefore, leave the field unchanged. Such motions occur if an entire magnetic field line interchanges its position with another line. Therefore, if the flow pattern is known on one plane intersecting the magnetosphere, the flow pattern of any other plane can be obtained by tracing the field lines to the second plane. We may also note that, since we have already satisfied momentum balance by the force-free condition, the determination of the flow pattern reduces principally to a kinematic problem. It also should be remembered that inside, but near the boundary as well as behind the earth where the field is weak, the foregoing restrictions do not apply since the plasma stresses can be appreciable.

Our knowledge of the flow of plasma within the magnetosphere is derived almost entirely from observations of the geophysical phenomena that this flow produces. There are two categories of geophysical phenomena which can be interpreted as resulting directly from plasma flow within the magnetosphere, namely, auroral motions and magnetic disturbances. The second of these is more completely understood because of the comparative ease of obtaining and reducing magnetic data. Also, since magnetic disturbance data may be represented by equivalent current systems flowing in the ionosphere,[37–39] the conservation of current allows the entire northern ionosphere currents to be mapped using simultaneous observations at a relatively few observatories. However, in the case of the aurora, to obtain a complete picture of auroral phenomena at a given time would require enough observatories to see the entire northern sky. Therefore, magnetospheric motions have been inferred mostly from magnetic disturbance data, and these motions have been shown to be consistent with auroral observations. It is well known that the field of ionospheric measurements and auroral observations is characterized by a wealth of detail that is described in many more or less subjective ways. We

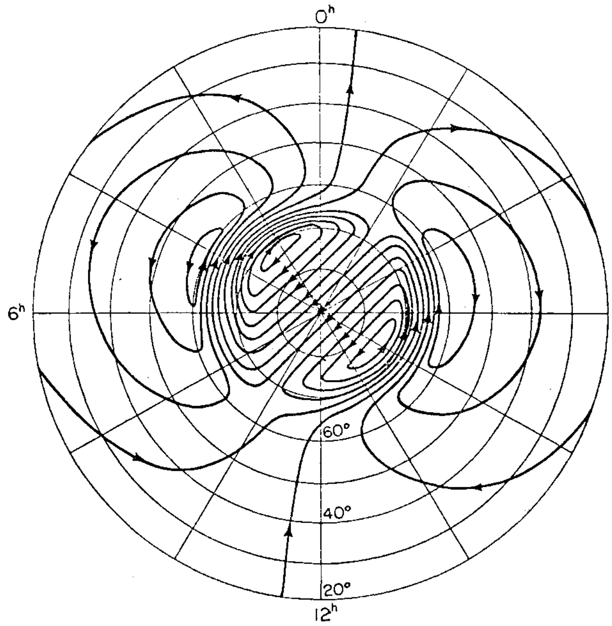

Fig. 1 Taken from Fukushima and Oguti.[39] Idealized ionospheric current system needed to produce the observed magnetic disturbance (*DS* component). 1.5 × 10⁴ amp flows between adjacent streamlines.

shall, in this paper, give only the barest outline of this mass of observational data and its interpretation.

The observed disturbances of the geomagnetic field are assumed to be due to currents flowing in the ionosphere. These ionospheric current systems have been derived for a number of magnetically disturbed days, and a model idealized system[40] is shown in Fig. 1. The pattern of motion of field lines in the ionosphere is the same as the current pattern, with the direction of motion opposite to the direction of current. This relationship between the field line motions and the ionospheric currents derives from two properties of the ionosphere, namely, the dominance of the Hall conductivity and the immobility of ionospheric ions. Since $\omega\tau$ for the electrons is large, the electron motions are frozen to the magnetic field lines, $E + v_e \times B = 0$. The ions, on the other hand, are immobile because of frequent collisions with the neutrals. This implies that the ionospheric currents are primarily due to the motions of electrons. Combining these properties gives the approximate relation $J = -neV_B$, where J is the ionospheric current, n the electron density, and V_B the velocity of the magnetic field lines. One can now see the "typical" motion of magnetic field lines by again referring to Fig. 1. The field line motions in the plane of the ionosphere are along the current streamlines, but in a direction opposite to that indicated by the arrows. If the current densities and electron densities are known, then the magnitude of the field lines' velocities can also be determined.

There appear to be two separate, but related closed patterns of motion of the field lines. One is a polar pattern in which field lines move across the polar region in the antisolar direction and return in a lower latitude band. This pattern is tilted to the west of earth-sun line. The second pattern is at lower latitudes and is composed of two separate closed systems, one on the morning side and one on the evening side of the earth. Each of these two subsystems rotates in a sense opposite to the adjacent polar system.

Next, we look briefly at the extent to which these derived motions of the field lines are supported by auroral observations. The plasma in the magnetosphere is frozen to the magnetic field. Hence, field lines and plasma move together, and this motion will be reflected in the motion of associated visual or radio auroras in the ionosphere. Observations of the motion of auroras[41] reveal that in the auroral zone (60°–

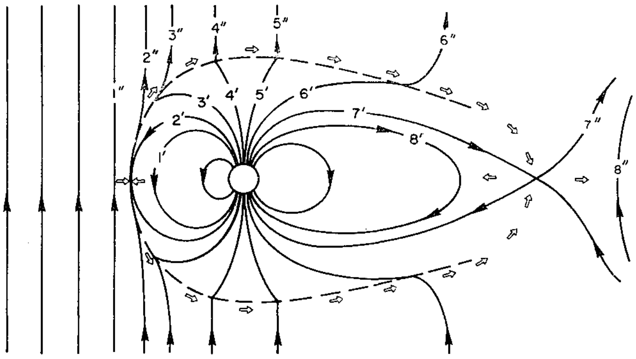

Fig. 2 Magnetic field configuration (heavy arrows) and plasma flow pattern (open arrows) allowing for some reconnection of field lines. Note that two neutral points are required since, for the whole flow field, the net rate at which interplanetary field lines become attached to dipole lines must be zero. Numbers indicate the motion of individual field lines with the motion progressing toward higher numbers. The figure is drawn in the plane determined by the earth-sun line and the earth's dipole axis. Details within the shock layer have not been shown.

70° geomagnetic latitude) auroras move to the west in the evening and to the east in the morning. At higher latitudes, the reverse situation occurs. This can be restated as: below approximately 70° geomagnetic latitude auroras move toward the sun, and above this latitude auroras move away from the sun. This is seen to agree qualitatively with the polar pattern deduced from magnetic disturbances. The crossover between eastward-moving auroras and westward-moving auroras in the auroral zone has been observed by Davis[42] to occur at approximately local magnetic midnight. This is about 3 hr later than one would expect from the ionospheric current pattern. The difference might possibly be due to the earth's rotation, which produces an apparent westward motion of the auroras which is comparable in magnitude to the observed auroral velocities. However, this point is not clear. Motions corresponding to the subpolar pattern show up in studies by Davis[43] of large auroral loops opening to the west near midnight. Motions of irregularities along a loop are clockwise if viewed from the ground, although the loop as a whole is moving westward partly because of the earth's rotation. This clockwise motion of auroras indicates a counterclockwise directed current that corresponds to the subpolar system.

Several theoretical models have been constructed in which the high latitude currents and auroral motions are generated in some manner, directly or indirectly by the action of the solar wind on the geomagnetic field. In the model of Dungey,[1] the interplanetary magnetic field plays a dominant role. He considers a situation in which the interplanetary field has a generally southward orientation. It will then combine with the earth's field to produce a neutral point in the subsolar region and also one in the antisolar region. The resulting steady-state magnetic field and plasma flow configurations in the plane containing the earth's dipole and the solar-wind direction are shown in Fig. 2. The field lines are indicated by continuous lines with heavy arrow heads giving the sense. The plasma motion is indicated by the open arrows; these arrows also indicate the motion of the field lines since the field is frozen to the plasma. (This figure omits some flow and field discontinuities that are not relevant at this point.)

Since the situation is assumed to be steady, the diagram can be interpreted as being either the total field topology and flow morphology at a given time or a time sequence of the motion of an individual field line and a "particle" of plasma. This second interpretation is made explicit in the diagram by labeling the order of events with numbers increasing with time. The subsolar apex of the field line whose initial position coincides with the line labeled 1′ moves sunward until it reaches the subsolar neutral point where field line cutting occurs (which will be discussed in Sec. V). Here the internal field line connects to the external field line, which has moved to the neutral point from an initial position which coincides with the line labeled 1″. The field lines now coincide with the position of the lines labeled 2′ and 2″. The lines indicated by 3 through 6 coincide with the positions reached at progressively later times by our two original field lines that now are joined and are being swept back by the solar wind. For a steady flow, the rate at which field lines become joined must equal the rate at which they become detached. Thus, eventually they must reach a position coinciding with line 7 (which intersects the antisolar neutral point) and separate. The internal field line then moves toward the earth 8′, and the external field line is carried away by the solar wind 8″. In order to complete the cycle the field line must move from position 8′ to position 1′. It is reasonable to suppose that this is accomplished by moving some distance towards the earth and then going around the earth.

The resulting motion of field lines in the ionosphere corresponding to the sequence 1–8 is seen to be across the poles and in an antisolar direction (assuming that the wind comes from a direction near the sun). This corresponds exactly to the solar-directed currents in the polar current system. As the field line returns from position 8 to position 1, its motion in the ionosphere must correspond to the lower latitude return band of the polar current system. The ability of this model to reproduce the general features of the polar current system is not critically dependent on the orientation of the interplanetary magnetic field. As long as some joining of field lines occurs, the field lines will be dragged in a generally antisolar direction across the poles and return at lower latitudes.

Hines and Axford[2] have greatly extended the description of the internal flow patterns in the magnetosphere. They observe that the correlations with observational data that they obtain are independent of the driving mechanism of the convection pattern. Although they list several possible mechanisms, including field reconnection, they tend to favor the existence of a shear stress at the magnetosphere boundary. More recently Axford[44] has observed that it would not be unreasonable for a transport of momentum across the boundary by waves to give the required shear stress. In addition to considering the convection pattern due to the drag of the magnetosphere boundary, they also consider the effects due to the rotation of the earth. Since the feet of the field lines move around with the earth, there will be some superposed rotational motion. In this manner they are able to account for some of the observed evening-morning asymmetries of high latitude geophysical phenomena such as auroral breakup, magnetic bays, and numerous ionospheric disturbances. The inclusion of the rotation also allows them to estimate the latitude at which the polar current pattern returns. They further postulate that the subpolar current system corresponds to a secondary convection zone driven by the primary convection zone, which in turn is driven by the solar wind.

This brief description does not do justice to the many details of the flow pattern and correlations with observations that Hines and Axford have succeeded in enumerating. We have, however, tried to emphasize that their model is based primarily on convection cells driven by an interaction with the solar wind. It is interesting to note that both they and Dungey felt that the evidence for such an interaction was strong enough to require them to postulate its existence even though no mechanism was known which required such an interaction. In Secs. V and VI of this paper we will observe that the rate at which field lines are reconnected at the boundary provides an interaction that is of the right order of magnitude to drive the primary convection cell. Thus, reconnection of field lines is probably the driving mechanism for the internal flow.

V. Boundary

Thus far in considering the external flow between the magnetosphere boundary and the shock wave we have been able to assume the plasma to have infinite conductivity. Within this approximation, the magnetosphere boundary would be an infinitely thin layer containing a current sheet which gives rise to the required change in magnetic field across it. If we now consider a finite, but high conductivity, this change in field will take place across a narrow but finite thickness. In this section we will briefly review an analysis[3] of this boundary layer, paying particular attention to the role of the normal component of the magnetic field across the layer which gives rise to the coupling between the external and the internal flows.

Let us first notice that we expect an appreciable change in the direction of the magnetic field across this boundary. The external magnetic field will be forced against the boundary by the wind. It will, therefore, lie essentially in the the plane of the boundary, but can have an arbitrary direction in this plane determined by its direction in the freestream. The internal magnetic field will also lie essentially in the plane of the boundary, but will have its direction determined by the dipole axis of the earth. Since these two directions are unrelated, we would, in general, expect to have an appreciable change in the direction of the magnetic field as we cross the boundary. Furthermore, since the magnetic field in the interplanetary plasma is known to fluctuate considerably, we would expect to have different changes in direction at different times. The simplest case to analyze is the case in which the direction of the magnetic field changes by 180°, that is, the field direction in the interplanetary plasma and the magnetic field of the earth are antiparallel. We will consider only this case and assume that the result is not too angle-sensitive and therefore that the results will apply approximately to any case in which the angle is large.

A sketch of the flow and magnetic field configuration of the over-all flow, including some resolution of the boundary layer, is shown in Fig. 3. The field lines in the interplanetary plasma are carried toward the magnetosphere by the solar wind. As the field lines cross the shock, they will be somewhat distorted, but they will continue to move toward the magnetosphere boundary. The field lines will reach the boundary first in the neighborhood of the stagnation point. Since the magnetic field is of opposite sign on the two sides of the boundary, there must be a point within the boundary where the magnetic field is zero. At such a neutral point in the magnetic field it is possible to have magnetic field lines cross. The motion of the magnetic field lines toward the boundary corresponds to an electric field perpendicular to the plane of the paper. Since for a steady flow $\nabla \times E = 0$, this electric field cannot change across a thin boundary and is therefore the same on both sides of the boundary. However, since the magnetic fields are in opposite directions, they move in opposite directions on the two sides of the boundary. In other words, the magnetic fields on both sides of the boundary move toward the boundary. When these field lines move through the crossed configuration at the neutral point they can reconnect themselves and move out along the boundary. That is, the top half of the interplanetary field line becomes connected to a field line that goes to the North Pole of the earth while the other half of the interplanetary field line becomes connected to the portion of the earth's field line coming from the South Pole. This is most easily visualized on Fig. 3 by considering the successive field lines drawn as a time sequence in the history of an individual field line. (This particular feature is indicated more clearly in Fig. 2, although the details of the shock layer are not shown.) Now such reconnection of the field lines is only possible in a finite-conductivity medium. For an infinite-conductivity medium the condition $E + v \times B = 0$ applied at the neutral point would require zero electric field. Zero electric field in turn would mean that the magnetic field lines approach the neutral point at zero velocity and, therefore, that no reconnection occurs. In the presence of finite conductivity this restriction does not apply since the electric field does not have to vanish when the magnetic field vanishes.

The foregoing discussion of the motion of the field lines was based, to some extent, on the two-dimensional picture. In the actual three-dimensional case, a field line does not have to continue to move toward the neutral point and be reconnected, but can move out of the plane of the paper and go around the magnetosphere. In fact, the infinite-conductivity treatment of the shock layer assumes that all of the field lines must go around the body. The number of field lines which will be reconnected rather than going around the body depends then upon the efficiency of the reconnection process.

The flow in the neighborhood of a neutral point has received considerable attention in the literature.[45–47] In particular, Parker[47] has analyzed a steady-flow situation that is in some respects similar to the case of interest here. His model is based entirely on the diffusion of magnetic fields due to the finite electrical conductivity of the medium. The result obtained is quite close to the usual stagnation-point solution in ordinary aerodynamics, namely that the rate at which fluid flows into the boundary layer decreases as the square root of the appropriate Reynolds number. For the high electrical conductivity, which we expect in the interplanetary plasma, the rate at which the plasma and, therefore, magnetic field lines move into the boundary layer would, on this basis, be extremely small. This analysis, however, grossly underestimates the rate at which field lines can move toward the boundary and get reconnected.

In a magnetohydrodynamic fluid it is not immediately obvious that the boundary layer must spread only by diffusion processes. For example, we know that a shear discontinuity, which in the absence of a magnetic field would spread only because of viscosity, in the presence of a magnetic field will propagate along magnetic field lines as an Alfven wave. Since the wave-propagation speed is independent of the conductivity of the medium, the wave will spread the information of the shear discontinuity much more rapidly into the fluid at high electrical conductivities than the diffusion proc-

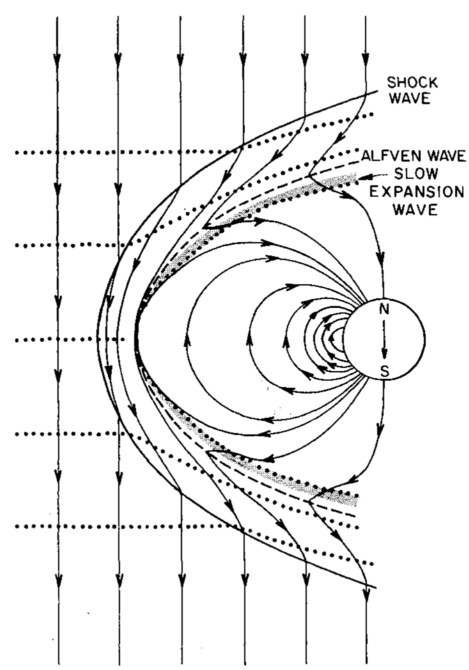

Fig. 3 Schematic drawing of magnetic field (solid lines) and plasma flow (dots) in the subsolar region. The flow is decelerated by the bow shock. The magnetosphere boundary is resolved into an Alfven wave and a slow expansion fan.

ess. We may therefore anticipate that there may be wave processes associated with the boundary which would spread the boundary much more rapidly than diffusion.

In a dissipationless magnetohydrodynamic fluid there are three different wave modes that can propagate. Although the flow behind the bow shock must be subsonic relative to the fast propagation speed, this flow can still be supersonic relative to the intermediate (i.e., Alfven) and slow propagation speeds. In such a case we might expect to find standing intermediate and slow waves somewhere in the flow behind the bow shock. Actually, the propagation speed of both the intermediate and the slow waves perpendicular to the magnetic field is zero. Therefore, precisely on the stagnation streamline we cannot have a standing wave at any flow velocity. However, if we go slightly away from the stagnation point along the boundary, the presence of field cutting at the stagnation point can give rise to a normal component of magnetic field (Fig. 3). Therefore, a short distance away from the stagnation point standing waves may exist. We may therefore look for a configuration in which the changes required across the boundary are accomplished by waves at some distance from the stagnation point, and diffusion is required only in the vicinity of the stagnation point. We might further expect that for very high conductivities the wave-propagation velocity can compete with the diffusion velocity for very small normal components of the magnetic field and therefore that the diffusion region can become very small. This reduction in the length of the diffusion region decreases its Reynolds number and therefore has the effect of allowing a much more rapid flow through the neutral point and hence an appreciably more rapid rate of field reconnection.

We must now ask whether it is possible to construct a set of waves which will take us from the external region, where there is a field of one sign and finite gas pressure, to the region behind the boundary, where the field is essentially of the opposite sign and where the gas pressure is zero. Such a set of waves can be found and are sketched in Fig. 3. The field reversal can be accomplished by an intermediate wave that has the property that the tangential component of magnetic field can be rotated through an arbitrary angle including 180°, although its magnitude does not change. The pressure and density of the plasma also do not change across such a wave. Behind the intermediate wave we are then left with a region in which the magnetic-field direction has already been reversed, but the gas pressure is still finite. We can now get from this condition to the zero pressure condition by means of a slow expansion fan in which the magnetic pressure increases to balance the decrease in plasma pressure. We may note that, as the fluid moves to the intermediate wave, the sharp bend in the magnetic field lines gives it a large acceleration. Thus, behind the intermediate wave the fluid is moving along the boundary at a speed roughly equal to the Alfven speed. This rapid motion represents a significant mass flow away from the stagnation region even in a thin layer.

In terms of rather gross arguments of momentum and mass conservation, it was shown[3] that such a wave region could be matched consistently to the diffusion region which is required in the neighborhood of the stagnation point. In fact, this matching can be done for a whole range of values of the velocity of the fluid approaching the stagnation point. As the velocity or the rate of field reconnection increases, the length of the diffusion region continually decreases. The existence of a range of values can be interpreted in terms of a low velocity flow over such a stagnation point. If the freestream flow velocity is within the range for which consistent solutions are allowed, then the stagnation streamline will not slow down as it approaches the stagnation point. The fluid coming in along this line can be pulled out fast enough by the acceleration across the intermediate wave. However, if the freestream velocity exceeds the maximum velocity allowed for by the boundary, the presence of the body will begin to affect the freestream. The stagnation streamline will have a velocity that decreases toward the stagnation point. The boundary will be at the point where the flow velocity just matches the maximum allowed velocity into the boundary layer. In the flow that we are considering, the freestream velocity is very high; therefore, the rate at which flow goes into the boundary layer will be the maximum one allowed by the rate of field reconnection.

The maximum rate at which field lines can flow toward the neutral point and become reconnected is determined by the fact that, in the wave region as the flow velocity increases, the normal component of magnetic field must increase. This flux through the boundary removes field lines from the stagnation region, thus decreasing the magnetic field in that neighborhood. As this field decreases, the rate at which field lines diffuse to the neutral point decreases, and therefore the entire process is self-limiting. Estimating this limit as occuring when the magnetic field has been reduced by a factor of 2 the following relation was obtained for the maximum flow velocity through the neutral point:

$$u_{\max} = \frac{V_A}{\ln(8\pi\sigma u_{\max}{}^2 R V_A{}^{-1})} = \frac{V_A}{\ln[(2u_{\max}/V_A)(R/\delta)]}$$

where $u_{\max}$ is the maximum flow velocity, V_A is the Alfven speed based on the absolute magnitude of magnetic field and the density in the region slightly upstream of the stagnation point, σ is the effective electrical conductivity in electromagnetic units, R is a scale length of the flow field along the boundary which can be taken roughly as the radius of the magnetosphere, and δ is the thickness of the boundary in the diffusion region. The foregoing relation was estimated to be good to about a factor of 2 if the electrical conductivity is known.

The significant feature of this result is that the rate of field reconnection depends only logarithmically on the electrical conductivity. Thus, even if the electrical conductivity is very high, the rate of field reconnection will be appreciable. The logarithmic dependence is also extremely fortunate in that it is quite insensitive to the considerable uncertainty in the value of the effective conductivity in the turbulent medium which was indicated in Sec. II.

The foregoing equation was written also in terms of the thickness of the diffusion region since it is probably easier to estimate this thickness than it is to estimate the conductivity itself. In order to obtain a lower limit to the rate of field reconnection, we may assume that the boundary-layer thickness in the diffusion region cannot be any smaller than an electron gyroradius. If it were smaller than this and carried the required current, the electrons would have to move at a velocity higher than their thermal speeds just ahead of the boundary layer. Such a thickness would seem to be a lower limit to the possible thickness of the boundary. Taking the magnetosphere radius as 10^5 km and the electron gyroradius as 3 km, the velocity of reconnection becomes $\sim 0.1\ V_A$. Probably a more reasonable value for the minimum thickness of the diffusion region would be the ion gyroradius, $\sim$ 100 km. Although some satellite evidence suggests this order of magnitude of thickness, the interpretation is somewhat ambiguous because of possible motion of the boundary. Using 100 km for the thickness of the diffusion region, we obtain a reconnection velocity of $\sim 0.2\ V_A$. We therefore conclude that the rate of field reconnection at the magnetosphere boundary is at least 0.1 V_A and is probably more like 0.2 V_A.

From this rate of reconnection we may now estimate what fraction of the field lines that pass through the bow shock wave also pass through the reconnection region rather than going around the magnetosphere without being cut. For a freestream ratio of flow speed to Alfven speed of 8, the flow velocity behind a normal shock is 1.1 V_A. The fraction reconnected is roughly the ratio of the flow through the stagnation point (uB) to that through the shock. If we assume that neither the magnetic field strength nor the Alfven speed change appreciably as we move from the shock toward the stagnation point, this would imply that the fraction reconnected is ~ 0.1 or 0.2, depending upon the choice of δ. Actually, the variations in the density and the magnetic field strength mentioned in Sec. III are in such a direction that this is an underestimate of the fraction of the field lines which will become reconnected. The lower limit to the fraction reconnected is therefore 0.1. There is, of course, an absolute upper limit of unity. As a result, the estimate of 0.2, which corresponds to a more likely choice of δ, is fairly closely bracketed.

The estimate of 0.2 corresponds to a significant fraction of the incident field lines becoming reconnected to the earth's field lines. In fact (Sec. VI), it is sufficient to drive the internal convective pattern. However, most of the field lines still go around the magnetosphere. Therefore, analyses of the external flow which neglect field reconnection are not significantly disturbed at least in the stagnation region, but significant effects may occur toward the wake.

VI. Over-All Flow

The estimate made in the previous section of the rate of field reconnection leads to the possibility of quantitative evaluation of various couplings between the internal and external flows. In addition to the transfer of momentum due to the dragging of the field lines by the solar wind, the existence of a normal component of magnetic field allows the flow of some plasma and high energy particles across the boundary. In the present paper we will, however, restrict ourselves to the discussion of two features. First, we will show that the quantitative estimate of the fraction of field lines reconnected leads to the right order of magnitude of auroral velocities from which it follows that the reconnection rate is sufficient to drive the internal flow. Secondly, we will discuss the relation of field cutting to the predictions of the magnetosphere shape behind the earth and Explorer X measurements in this region.

As discussed in Sec. IV, the over-all flow patterns observed in the auroral regions are consistent with the picture of some reconnection of field lines. In order to claim that field reconnection is responsible for driving this internal flow, we must still show that it produces quantitatively the correct order of magnitude of flow. This can be done most directly by comparing the total flow rate of magnetic field lines. The flow rate of field lines within the polar convection pattern must equal the flow rate of magnetic field lines that intersect the magnetosphere boundary. The rate at which field lines are incident on the bow shock wave is the product of the interplanetary magnetic field, $\sim 5 \times 10^{-5}$ gauss, the flow velocity of the solar wind, ~ 500 km/sec, and the diameter of the magnetosphere which we will take as 20 earth radii, or about 10^5 km. From our estimate that one-fifth of these field lines become reconnected, the flux of field lines that are joined to the earth is about one-fifth of this product, or approximately 500 gauss-km^2/sec (or 5×10^{12} Maxwells/sec). The flow rate in the ionosphere in the polar regions is the product of the local magnetic field strength, 0.6 gauss, the width of the region in which the flow is in one direction, which can be seen from Fig. 1 to be about 30° or 3×10^3 km, and auroral velocities that are typically between 0.1 and 0.5 km/sec.[41]‡ This gives a total flow rate of 200 to 1000 gauss-km^2/sec. The estimated flow rate at the magnetosphere boundary is therefore in excellent agreement with the observed flow rates in the ionosphere. Thus, field reconnection not only gives qualitatively the current pattern observed in the ionosphere, but also gives a quantitatively correct prediction of the total flow rate involved.

It has been suggested that the Explorer X measurements give direct evidence for the existence of a normal component of magnetic field at the boundary of the magnetosphere.[34, 35] The orbit of this satellite had an apogee of about 40 earth radii in an antisolar direction. Data were recorded only during one outward pass. In this time the satellite apparently crossed the boundary of the magnetosphere several times most likely because of the motions of the boundary. At each transition the average magnetic field was observed to change both in direction and in magnitude corresponding to a transition between an internal and an external field. Plasma probe measurements also indicated regions in which there was and was not plasma, the boundary being consistent with the magnetic field indication. If a physical boundary is drawn at each transition consistent with the position of the satellite and a reasonable wind direction, then the measurements of the magnetic field direction indicate a substantial normal component of the field to the boundary. This argument has been criticized on the basis that the direction of the boundary may not be known with sufficient accuracy to make the measurement of the direction of the magnetic field relative to the boundary significant.[35]

In addition to the foregoing argument, which may be open to some question, Explorer X provided other evidence that can be interpreted in terms of escaping field lines. The internal magnetic field strength near the boundary dropped appreciably more slowly than a dipole field as the satellite increased its distance from the earth. Thus, at 30 earth radii, the field strength was about 15γ. If the field lines did not escape and currents were restricted to the magnetosphere boundary, then the image dipole method could be used to estimate the strength of the internal field. This procedure would tend to give a reduction in the field strength at these distances from the earth as compared to the dipole field strength. Hence, the field strength at 30 earth radii should be less than the corresponding dipole field strength which is about 2γ. The observed field strength is therefore almost 10 times too large at this point. This increased field strength is consistent with the observation that the cavity appears to be conically expanding and therefore makes a larger angle with the wind direction than would be expected from the theoretical analyses. These data indicate that some stress must exist which drags the field lines with the wind and therefore increases the field strength in these regions. It is tempting to suggest that this stress is provided by the field lines that escape through the magnetosphere boundary. If this can be found to agree quantitatively it would indicate that the field reconnection can also give rise to significant changes in the shape of the magnetosphere.

Appendix: Early Results from the Satellite IMP

Since this report was prepared, a considerable amount of information received from the IMP satellite (Explorer XVIII) has become available. This satellite was launched into a highly eccentric orbit (apogee ~ 30 earth radii) on November 27, 1963. Since then it has monitored magnetic fields[48] and particle fluxes[49, 50] through a large part of the magnetosphere. The results provide direct confirmation of several features of the magnetospheric flow which had been predicted, but which had not hitherto been observed. IMP has already proven to be a powerful tool for the study of the magnetosphere, and there can be no doubt that the analysis of all the information received will provide us with a greatly increased understanding of the charged particles and magnetic fields in the environs of the earth. Though all this analysis is still far from complete, the results published thus far have a considerable bearing on this report, and we are therefore pleased to adopt the suggestion of a reviewer and

‡ The flow rates derived from ionospheri ccurrents are not as well determined as auroral velocites because of uncertainties in the electron density in the ionosphere. However, within these uncertainties, they agree.

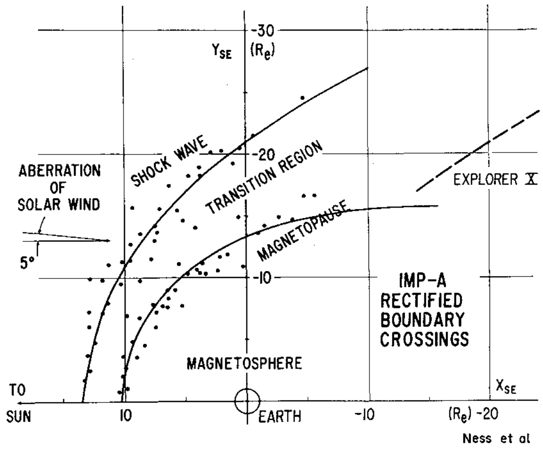

Fig. 4 This figure, reproduced from Ness, et al.,[48] shows the shape of the over-all flow in the magnetosphere as established by measurements of the magnetic field. The origin of the coordinate system used is the center of the earth. The X axis is the earth-sun line; the Y axis lies in the ecliptic plane, and the units on both axes are earth radii. The solid lines represent the positions of the shock wave and boundary as calculated theoretically by Spreiter and Jones[31] and others. These calculations have been modified to allow for the diurnal change in the tilt of the earth's magnetic axis and the aberration effect due to the motion of the earth in orbit. The data represent the experimental points; since these were generally not in the plane of the eclipitic, each point was rectified by rotation in a meridian plane. Shown for comparison is the orbit of Explorer X,[34, 35] similarly rectified.

include here a brief critical summary of the information presently available (August 1964).

In the first place, the existence of a clearly defined over-all flow pattern consisting of a shock wave, a transition region, and an interface (boundary) has been amply demonstrated. Both the shock and the interface are thin compared to their separation distance, thus substantiating the arguments given in Sec. II for the validity of the continuum analysis. The observed flow pattern is in excellent agreement with general predictions as to both its nature and its shape. The positions of the shock and boundary as determined by the magnetic data are shown in Fig. 4 (reproduced by kind permission of Ness, Scearce, and Seek). The agreement with aerodynamic theory (that of Spreiter and Jones[31] and others) is seen to be excellent, especially when it is recalled that the data shown was collected over a period of about 3 months during which time (in spite of a low level of solar activity) the velocity and density of the solar wind could hardly have remained perfectly constant.

The magnetic field and particle measurements demonstrate clearly the existence of a collisionless shock wave. Across this shock, the plasma flow changes abruptly from supersonic to subsonic as evidenced particularly by the directional resolution of the plasma probe.[49] Ahead of the shock the flow is highly directed, whereas behind the shock the flux is approximately isotropic. The magnetic field increases sharply across the shock and also changes direction. Figure 5 (also reproduced from Ness et al.)[48] shows the magnetic field strength and variance as measured on a single pass. The shock is clearly visible at about 22 earth radii. It is apparent that the field becomes markedly noisier behind the shock. This presumably corresponds to turbulence that was produced within the shock in order to accomplish the necessary dissipation.

At the present time the data have not been analyzed in sufficient detail to determine the actual shock thickness or to define clearly the nature of the turbulent dissipation process. Some indications exist, however, which suggest that the important scale length is the ion gyroradius as proposed in Refs. 12–14, rather than the smaller scale lengths proposed in Refs. 8–10. In Fig. 5, although the magnitude of the field appears to rise abruptly in less than the time between adjacent plotted data points, the variance rises over a measurable distance ($\sim 10^3$ km) corresponding roughly to an ion gyroradius based on the velocity and magnetic field in the supersonic stream. In data from some other passes, the rise in magnitude of the magnetic field also occurs over a measurable distance of the same order of magnitude. Also, the observation of a variance in the field comparable to the average field based on an averaging procedure which did not include frequencies above the ion cyclotron frequency suggests that significant turbulence resides in the frequency range of the ion cyclotron frequency or below. Since these frequencies correspond to wavelengths of the order of the ion gyroradius or longer, the basic scale of the turbulence is probably in this range. Since the raw data contain an appreciably better frequency response that is represented by the averages that have been published thus far, we may anticipate that future analysis will provide a much clearer definition of the properties of collision free shock waves. One rather surprising observation is the apparent existence[50] of a thin (< 500 km) region containing electrons with energies greater than 30 kev. Although the possibility of heating electrons in a collision free shock and producing a uniform region of energetic electrons behind the shock had been previously discussed, the possibility of the existence of a narrow spike had not been anticipated.

The evidence of IMP on the question of field cutting and reconnection as discussed in this report is not yet clear. Di-

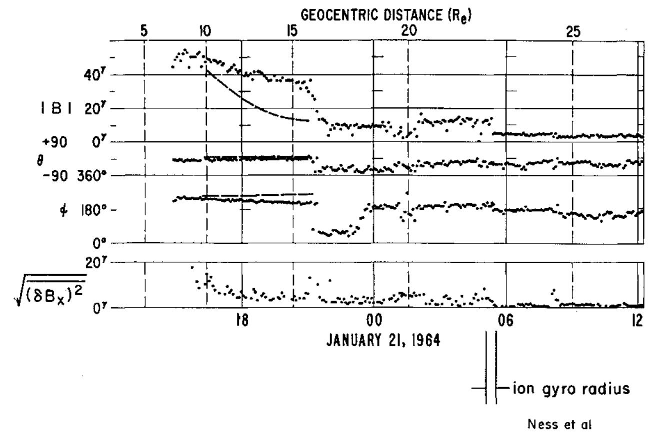

Fig. 5. These data, also from the report of Ness, et al.,[48] show the strength of the magnetic field and a measure of its noisiness as recorded by the satellite on an outbound pass. The angles Θ and ϕ give the direction of the magnetic field. Θ determines a latitude; Θ = 0 defines the ecliptic plane. ϕ determines a longitude, $\phi = 0$ is the direction towards the sun. The dashed lines represent an extrapolation of the earth's magnetic field. The compression of this field to higher values as the boundary is approached, and the rapid drop at the boundary ($\sim$ 16 earth radii) is clearly visible. Although these data could be interpreted as supporting our prediction that the field should change direction (Alfven wave) ahead of the place where it increases in magnitude (slow expansion), the point is not very clear. Most of the other passes illustrated are even less clear, so that this question can not yet be regarded as settled. Ahead of the boundary, the field strength remains on the order of 10 γ, with a noise level not much smaller, indicating considerable turbulence until ($\sim$ 22 earth radii) the satellite crosses the shock wave. After that, the instrument records the very steady interplanetary magnetic field of a few γ's. The variance in this region is equivalent to the noise level of the instrument and the analytical procedure. The spike in the noise at $\sim$ 24 earth radii appears on several passes and has been interpreted[48] as a precursor wave.

rect measurement of the normal component of the magnetic field at the boundary is probably difficult since the predicted value is only about 10% of the tangential component. However, if the wave pattern illustrated in Fig. 3 is indeed present, it should be the case that the field changes direction at one place (Alfven wave) and changes strength (slow expansion) at another. The spatial separation between these two changes depends upon the distance along the boundary away from the neutral line at which the measurement is made. Since the position of the neutral line is not known, the expected separation is not clearly defined. The boundary that in Fig. 5 occurs at about 16 earth radii shows a change in both magnitude and direction; however, it does seem to show a small separation between them, but this is not entirely clear; other passes were even less clear. A small fraction of the boundary crossings observed by Cahill§ with Explorer XII did show such a separation, the majority, however, did not. A clearer resolution of this point must await a more careful analysis of the data.

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DECEMBER 1964 AIAA JOURNAL VOL. 2, NO. 12

Small Magnetofluid-Dynamic Peristaltic Motions Inside an Annular Circular Cylindrical Induction Compressor

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The small magnetofluid-dynamic peristaltic motions inside an annular circular cylindrical induction compressor are studied. The compressor consists of a circular cylindrical traveling wave tube of radius r_0 on which is impressed a purely sinusoidal current sheet of the form $nI \exp i(kz - \omega t)\hat{\theta}$, where nI is the number of ampere turns per unit length, k the wave number, and ω the circular frequency. Inside the tube, an annulus $r_0 \geq r \geq r_1$ is filled with a highly conducting fluid that is constrained at the ends not to move in the axial direction. The fluid to be pumped moves in an annulus $r_1 \geq r \geq r_2$ and is separated from the conducting fluid by an impermeable flexible diaphragm. The electromagnetically induced motions of these two inviscid and incompressible fluids, when the electromechanical coupling is weak, i.e., in the limit of small magnetic Reynolds number (based on wave speed and wavelength), are examined analytically. From the nature of the axial component of the body force induced in the constrained conducting fluid, it is shown that a time-averaged constant axial pressure gradient is induced which is transmitted to the pumped fluid by virtue of the mechanical coupling between them. If now, the induced, purely oscillatory, radial force component is sufficient to pinch and trap the pumped fluid, the ensuing motion of the latter would consist of trapped packets of fluid traveling in the wave direction with the wave speed of the diaphragm (equal to the speed of the traveling current sheet) against the pressure gradient induced in it.

I. Introduction

RECENTLY, a great deal of interest has been shown in the application of the principles of magnetofluid-dynamics to the compression and acceleration of poorly conducting liquids (e.g., sea water), with an eye toward its possible application to the propulsion of undersea craft.[1, 2] To date, most of all of the schemes proposed involve the direct interaction of the magnetic and electric fields with the poor conductor. Since the electrical conductivity of sea water is approximately six orders of magnitude less than ordinary liquid metal conductors, such schemes are doomed to failure because the magnitudes of the induced currents are so small that tremendous field strengths are required to produce significant forces. However, this low conductivity limitation could be circumvented, for example, if a pumping scheme could be devised wherein the electromagnetic field, instead of acting directly on the poor conductor, is made to act directly on an intermediate working fluid of large conductivity. If now, the working fluid and the fluid pumped are mechanically coupled, the large forces induced in the working fluid could conceivably be transmitted to the pumped fluid.

A possible scheme that utilizes the foregoing concept is shown schematically in Fig. 1. An annulus $r_0 \geq r \geq r_1$, inside a circular cylindrical tube of radius r_0, is filled with a highly conducting fluid that is constrained at the ends not to move in the axial direction. The pumped fluid, moving in the annulus $r_2 \leq r \leq r_1$, is separated from the conducting fluid by a flexible impermeable diaphragm. Impressed on a transmission line, in the form of a coil wound around the cylinder, is a purely sinusoidal traveling current sheet of the form $nI \exp i(kz - \omega t)\hat{\boldsymbol{\theta}}$, where nI is the amplitude of the number of ampere turns per unit length, k the wave number, ω the circular frequency, and $\omega/k = V$, the wave speed. If the relative speed between the fluid conductor and wave is different from zero, then from the circular symmetry, closed azimuthal currents will be induced in the conductor. These currents, when crossed with the radial and axial components of the traveling **B** field associated with the traveling current sheet, produce axial and radial body forces, respectively. Since the conducting fluid is constrained at the ends not to move in the axial direction, the axial velocity of the fluid

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